UNIT 2

BINOMIAL THEOREM

Binomial Theorem

Binomial theorem is a mathematical formula used to expand two-term (binomial) expressions.

- The theorem states that $(a + b)^n$ can be expanded using the following expression:

$$(a + b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

Binomial Expansion

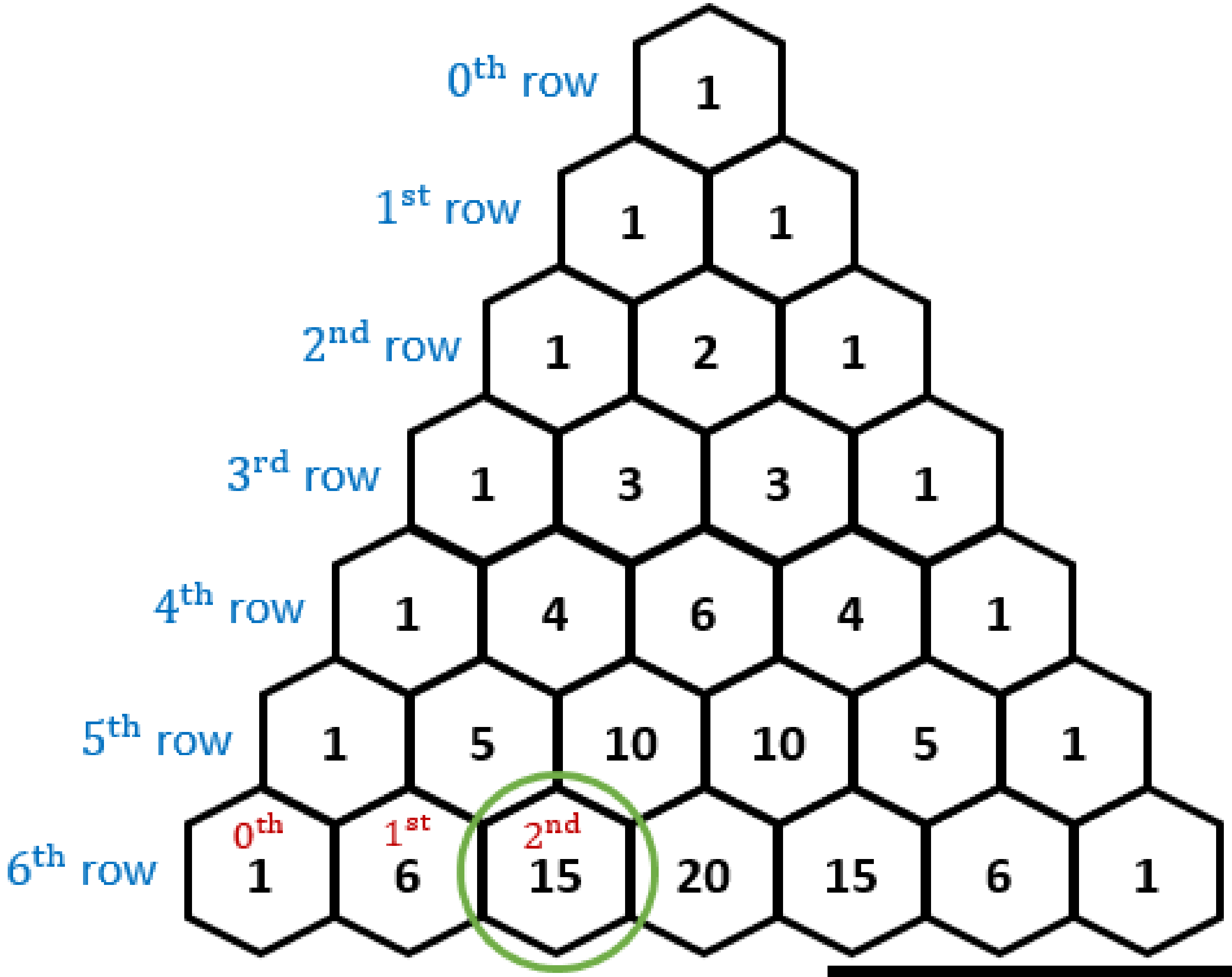
Binomial Theorem gives the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers while,

Pascal's Triangle can be used to determine the coefficients in this expansion

PASCAL'S TRIANGLE

Pascal's Triangle

Pascal's Triangle is a triangle displaying a pattern of numbers in which the terms in additional rows are found by adding pairs of terms in the previous rows, so that any given term is the sum of the two terms directly above it.



LET'S DRAW PASCAL'S TRIANGLE!

Binomial Expansion using Pascal's Triangle

Binomial expansion	Pascal's Triangle	Row
$(a + b)^0 = 1$	1	0
$(a + b)^1 = a + b$	1 1	1
$(a + b)^2 = a^2 + 2ab + b^2$	1 2 1	2
$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$	1 3 3 1	3
$(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$	1 4 6 4 1	4
$(a + b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$	1 5 10 10 5 1	5

The pattern in Pascal's Triangle can be used to **find the coefficients** of terms in the expanded form of $(a + b)n$. The coefficients of the terms depend on **the power of the binomial, *n***.

Binomial Expansion using Pascal's Triangle

<u>Steps:</u>

1. **Create Pascal's Triangle to the appropriate row.**
   - Note that the first line of the triangle is "row 0." For example, if the expression $(6x + 2y)$ is raised to the third power, the first four rows of the triangle are needed; that is, rows 0–3.
2. **Identify the row of Pascal's Triangle with the coefficients of the expanded expression.**
   - If the power of the binomial is 3, so the coefficients will come from row 3 of Pascal's Triangle.
3. **Write the expanded expression with the coefficients and powers of each term.**
   - The powers of the terms will follow the pattern a^3, a^2b^1, a^1b^2, b^3.
   - The coefficients of $(a + b)^n$ can be found in the nth row of Pascal's Triangle. .
4. **Evaluate each term.**
   - Use the order of operations to evaluate each term, first evaluating any exponents, then finding the products.

REMEMBER …

1. Get your coefficients from Pascal's Triangle.

2. The Exponent = 2nd number of Correct Row.

REMEMBER …

3. Exponents on the first term decrease.

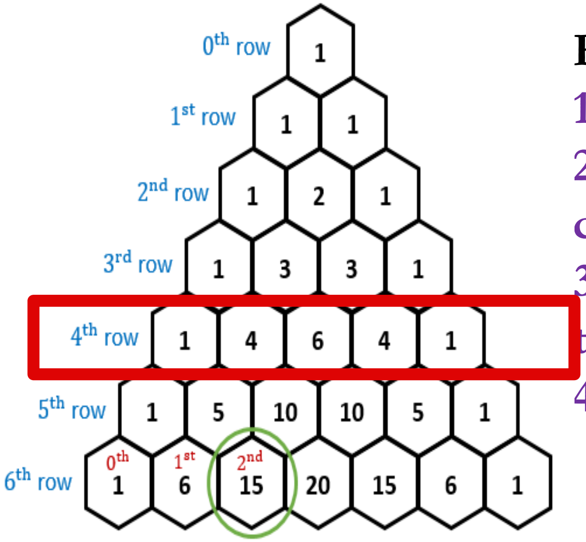
REMEMBER …

4. Exponents on the second term increase.

Example 1

Expand the following

$$(x+3)^4$$



Remember the steps:

1. Create Pascal's Triangle to the appropriate row.
2. Identify the row of Pascal's Triangle with the coefficients of the expanded expression.
3. Write the expanded expression with the coefficients and powers of each term.
4. Evaluate each term.

$$1x^4(3)^0 \quad 4x^3(3)^1 \quad 6x^2(3)^2 \quad 4x^1(3)^3 \quad 1x^0(3)^4$$

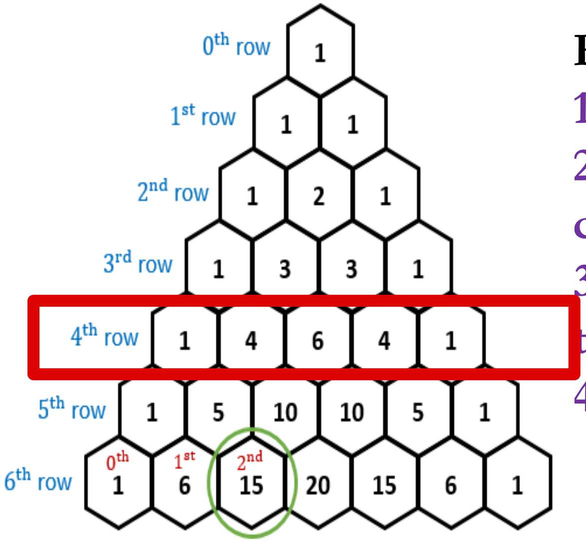
$$1x^4(1) \quad 4x^3(3) \quad 6x^2(9) \quad 4x^1(27) \quad 1(1)(81)$$

$$x^4 + 12x^3 + 54x^2 + 108x + 81$$

Example 1

Expand the following

$(p+4)^4$



Remember the steps:

1. Create Pascal's Triangle to the appropriate row.
2. Identify the row of Pascal's Triangle with the coefficients of the expanded expression.
3. Write the expanded expression with the coefficients and powers of each term.
4. Evaluate each term.

$$1p^4(4)^0 \quad 4p^3(4)^1 \quad 6p^2(4)^2 \quad 4p^1(4)^3 \quad 1p^0(4)^4$$

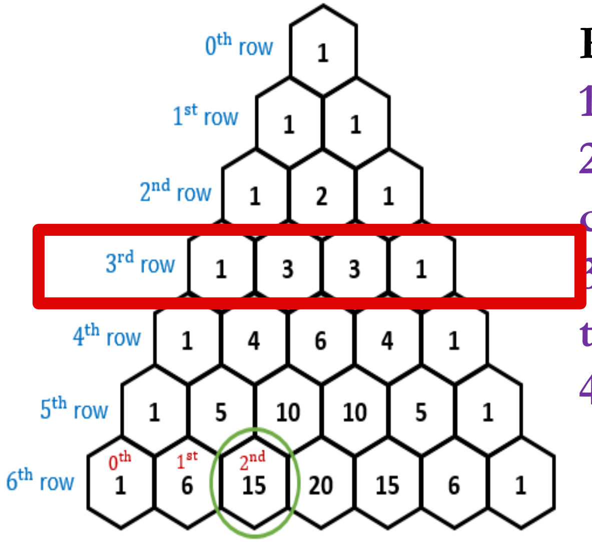
$$1p^4(1) \quad 4p^3(4) \quad 6p^2(16) \quad 4p^1(64) \quad 1(1)(256)$$

$$p^4 + 16p^3 + 96p^2 + 256p + 256$$

Example 1

Expand the following binomial:

$$(x - 2)^3$$



Remember the steps:

1. Create Pascal's Triangle to the appropriate row.
2. Identify the row of Pascal's Triangle with the coefficients of the expanded expression.
3. Write the expanded expression with the coefficients and powers of each term.
4. Evaluate each term.

$$1x^3(-2)^0 \quad 3x^2(-2)^1 \quad 3x^1(-2)^2 \quad 1x^0(-2)^3$$

$$1x^3(1) \quad 3x^2(-2) \quad 3x(4) \quad 1(1)(-8)$$

$$x^3 - 6x^2 + 12x - 8$$

Determine the coefficient described.

1. What would be the coefficient of the third term of $(p - 5)^6$?

2. What would be the coefficient of x^2y^2 of $(x + y)^4$?

Determine the coefficient described.

1. What would be the coefficient of the third term of $(p - 5)^6$?

375

2. What would be the coefficient of x^2y^2 of $(x + y)^4$?

6

Determine the terms described.

1. What would be the fourth term of $(x + 4)^5$?

2. What would be the third term of $(a – 3)^4$?

Determine the terms described.

1. What would be the fourth term of $(x + 4)^5$?

$$640x^2$$

2. What would be the third term of $(a - 3)^4$?

$$54a^2$$